Learning Probabilistic Programs

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Recap of Last Lecture

- Evaluation-based inference
  - Dynamic
  - Can deal with programs with unbounded loops

Likelihood Weighting

- A form of importance sampling where the proposal is the prior

$$\mathbb{E}_{q(X)}\left[\frac{p(X|Y)}{q(X)}r(X)\right] = \frac{1}{p(Y)}\mathbb{E}_{q(X)}\left[\frac{p(Y,X)}{q(X)}r(X)\right]$$
$$\simeq \frac{1}{p(Y)}\frac{1}{L}\sum_{l=1}^{L} W^l r(X^l),$$

$$W^l = \frac{p(Y,X^l)}{q(X^l)} = \frac{p(Y|X^l)p(X^l)}{p(X^l)} = p(Y|X^l)$$

If we use $p(X^l)$ as the proposal distribution

Y are observed/conditioned variables

Likelihood Weighting: Variants

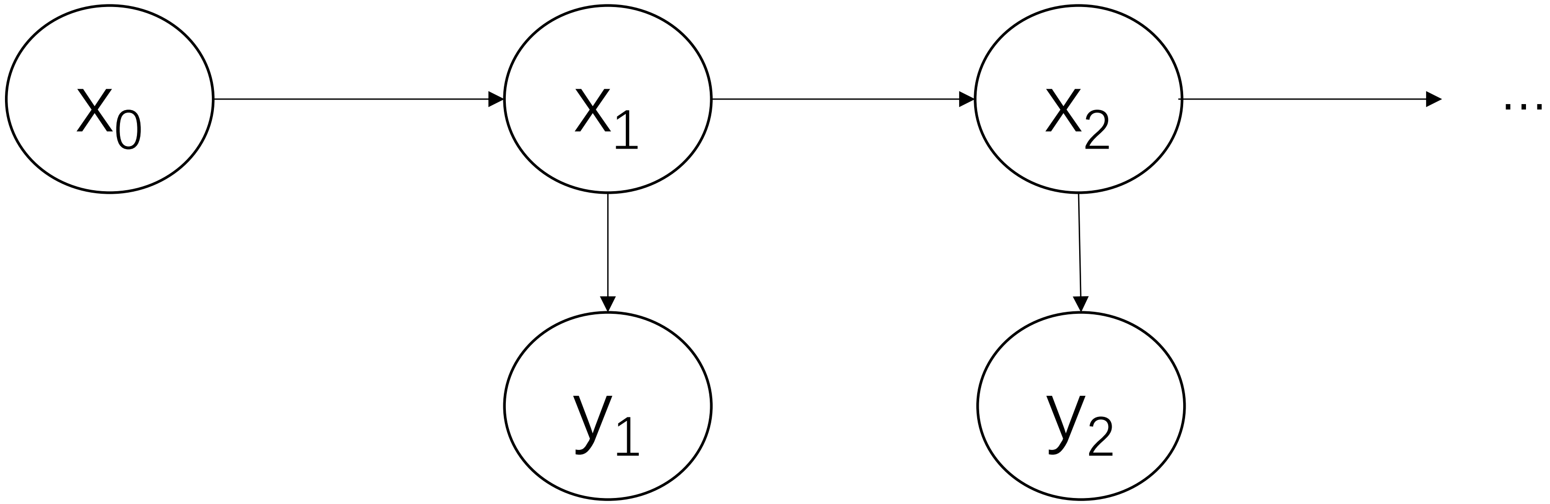
- Naïve Metropolis Hasting (draw random traces)

- Single-site proposal (try to only chance one variable at a time)

Sequential Monte Carlo

- In probabilistic programming, sample a high-dimensional distribution by sampling a sequence of lower dimensional distributions
- Also called particle filters
- Used in signal processing and probabilistic inference

SMC: Problem Statement



Given

$p(x_0)$ and
$p(x_t|x_{t-1})$ and
$p(y_t|x_t)$ and
Observations $y_{1:t}$

Estimate

$p(x_{0:t}|y_{1:t})$ or
$p(x_t|y_{1:t})$ or
$I(f_t) = E_{p(x_{0:t}|y_{1:t})}[f_t(x_{0:t})] = \int f_t(x_{0:t}) p(x_{0:t}|y_{1:t}) dx_{0:t}$

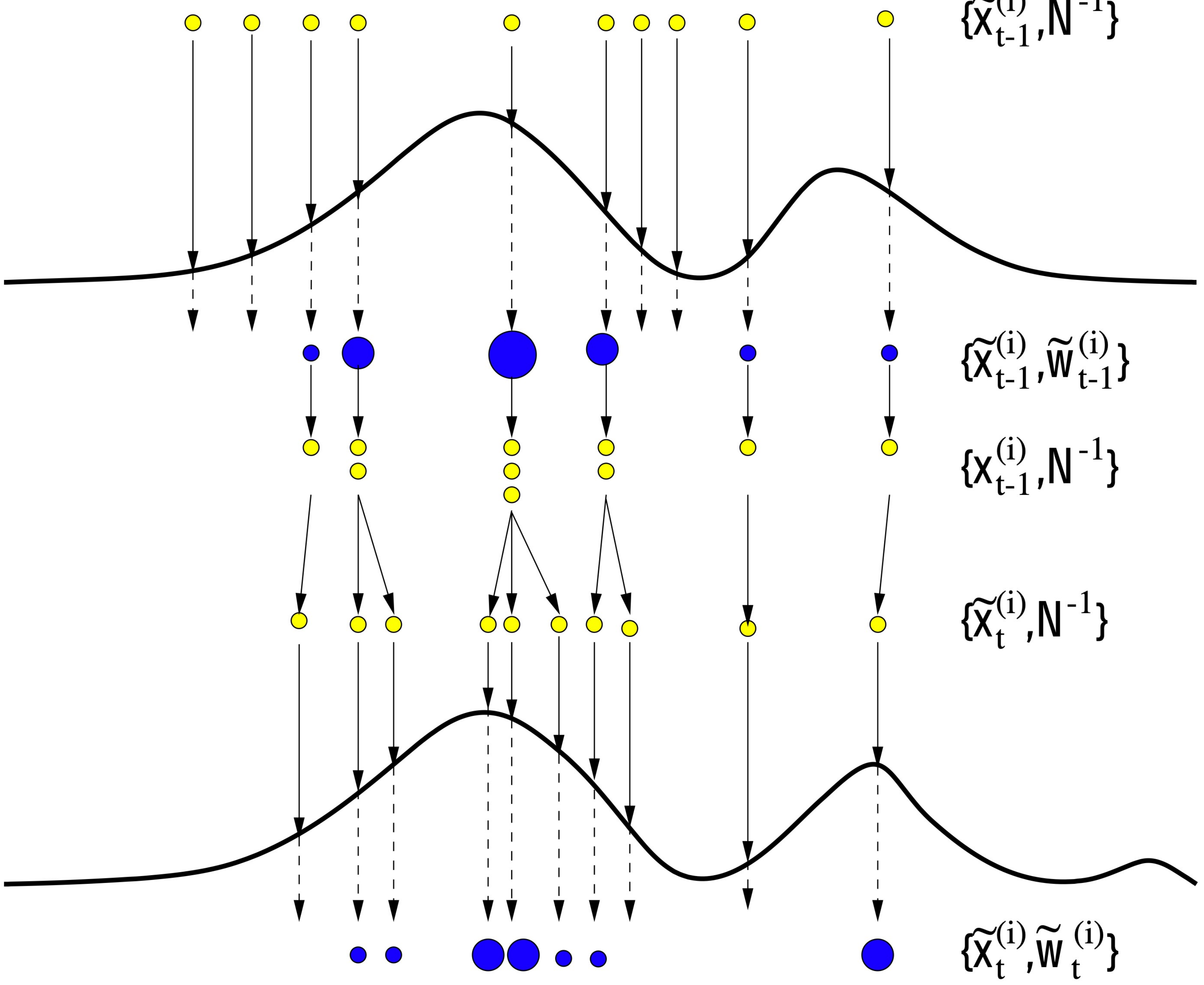
SMC: Main Ideas

- Sample on the Markov chain:

$$\pi\left(\mathbf{x}_{0:t} \middle| \mathbf{y}_{1:t}\right) = \pi\left(\mathbf{x}_{0:t-1} \middle| \mathbf{y}_{1:t-1}\right) \pi\left(\mathbf{x}_t \middle| \mathbf{x}_{0:t-1}, \mathbf{y}_{1:t}\right).$$

- Reweight the samples using importance sampling

- Throw away the samples (particles) with low probabilities i=1,...,N=10 particles



From “An Introduction to Sequential Monte Carlo Methods” by Arnaud Doucet, Nando De Freitas, and Neil Gordon

SMC: Bootstrap Filter

Assume the proposal distribution is $p(x_{1:t})$

1. Initialization. T = 0
   - For i = 1,…,N, sample $x_0^{(i)} \sim p(x_0)$ and set $t = 1$
2. Importance sampling step.
   - For sample $\tilde{x}_t^{(i)} \sim p(x_t | \tilde{x}_{t-1}^{(i)})$ and set $(\tilde{x}_{0:t-1}^{(i)}, \tilde{x}_t^{(i)})$.
   - For i = 1,…,N, evaluate the importance weights.
   - Normalize the importance weights
3. Selection step
   - Resample with replacement N particles from the current particles according to importance weights
   - Set $t \to t + 1$

Question 1

- In evaluation-based method, if the sampled trace doesn’t terminate, what would you do in practice?

Question 2

- Consider the program x = unform(0, 1); y = gaussian(x, 1). Suppose the current trace is x = 0.5, y = 1. Now we want to chance y, what is p(y) that we‘re sampling from?

- What if we want to change x?

Question 3

- Consider the program

```
x = 0;
while(bernoulli(0.5));
    x+=1
condition(x > 2)
```

- Describe an algorithm to sample traces from it.

Question 4

- Sequential Monte Carlo can be see as a variant of importance sampling. Is the statement right?

Question 5

- What would happen if we don't throw away particles in sequential Monte Carlo?

This Lecture

- Learning in probabilistic programming
  - Still an active research area
  - Not a solved problem

Question

- Can you define inference and learning?

Inference vs. Learning

- Inference: given $f|\theta$, run $f|\theta$ to output data

- Learning: given $f|\theta$, and data D, figure out θ

Inference vs. Learning

- Inference is often a part of learning
  - Example: perform inference with different parameters

Inference vs. Learning

- Inference is often a part of learning

```
p = bernoulli()
D = [….]
if p == 1:
        m = model1
else:
        m = model2

for (x,y) in D;
        condition(m(x)+N(0,0.1) == y)

output m
```

Learning in Probabilistic Programming

- Parameter learning

```
x = uniform(p1, p2)
y = gaussian(x, p3)
if(bernoulli(p4))
        z = x
else
        z = y
condition(z > 100)
```

What are p1, p2, p3, p4?

Learning in Probabilistic Programming

- Structure learning

```
x = uniform(p1, p2)
y = gaussian(x, p3)
if(bernoulli(p4))
        z = x
else
        z = y
condition(z > 100)
```

More on Structure Learning

- How to synthesize (deterministic) programs is an active field

- Program synthesis
  - Started early
  - Still under development
  - Works well in specific settings

Program Synthesis

- Given a specification, generates a program that satisfies the specification
- Main challenge: intractable search space
- Various approach to cut the search space
  - Sketch
  - SyGuS (Syntax-Guided Synthesis)

Program Synthesis: Sketch

```
if (x > ??)
        y = 100
else
        y = ??
output x*x+y*y
```

x = 1, o = 100

x = 10, o = 1000

Program Synthesis SyGuS

Syntax Constraints:

$$e \coloneqq input \mid e + e | e * e | e - e | e/e$$

Semantic Constraints:

$$e(2) = 100$$
$$e(5) = 700$$
$$\ldots$$

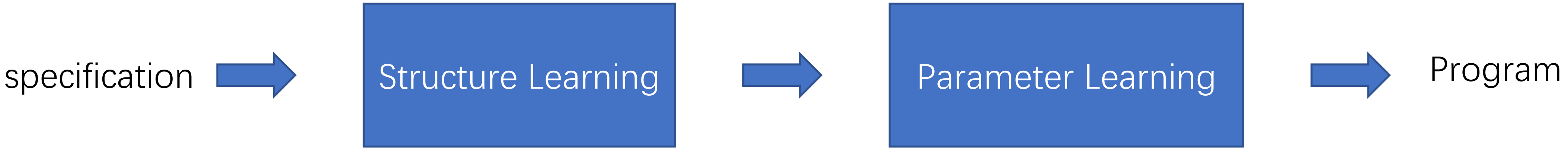
The semantics constraints can be more high-level than input-out examples. For example, the output of a sorting algorithm is sorted.

More on Program Synthesis

- https://people.csail.mit.edu/asolar/SynthesisCourse/TOC.htm

- https://xiongyingfei.github.io/SA/2020/main.htm

A Possible Pipeline to Synthesize Probabilistic Programs



Two Typical Approaches

- Non-Bayesian method (Maximum Likelihood)
  - Kevin Ellis, Armando Solar-Lezama, Joshua B. Tenenbaum: Unsupervised Learning by Program Synthesis. NIPS 2015.

- Bayesian method
  - Feras A. Saad, Marco F. Cusumano-Towner, Ulrich Schaechtle, Martin C. Rinard, Vikash K. Mansinghka: Bayesian Synthesis of Probabilistic Programs for Automatic Data Modeling. POPL 19.

Ellis et al., 2015: Motivations

- Goal: unsupervised learning
  - Induce good latent representations of a data set
- Programs are a natural knowledge representation for many domains
  - Compression: find smallest representation
  - Infer both programs and inputs
- General solution is head
  - Encode domain-specific parts using a DSL

Key Ideas

- Using PCFG to limit the program space

- Symbolic search: SMT

Problem Formalization

Minimize

$$\underbrace{-\log P_f(f)}_{\textbf{program length}} + \sum_{i=1}^{N} \left(\underbrace{-\log P_{x|z}(x_i|f(I_i))}_{\textbf{data reconstruction error}} \quad \underbrace{-\log P_I(I_i)}_{\textbf{data encoding length}} \right)$$

f is drawn from a prior determined by the sketch

I is drawn from a domain-dependent description length prior P_I, which leads to $z_i = f(I_i)$.
$P_{x|z}(* \mid z_i)$ estimates the error between predictions and observations.

Program is largely deterministic, but inputs are random. Also, going from z to x is a random process (manually specified)

Defining a Program Space

- Probabilistic context-free grammar (PCFG)
  - Place probabilities on production rules

$$\mathcal{E} \rightarrow \mathcal{E} + \mathcal{E} \mid \mathbb{R} \mid x$$

- Define denotations for each rule using SMT

$$[\![\mathcal{E}_1 + \mathcal{E}_2]\!](I) = [\![\mathcal{E}_1]\!](I) + [\![\mathcal{E}_2]\!](I) \qquad [\![r \in \mathbb{R}]\!](I) = r \qquad [\![x]\!](I) = I$$

- We can use SMT expression to denote the synthesis problem

Solution

- Construct an SMT that
  - Defines the space of programs
  - Computes the description length
  - Computes the output given an input and a program

- Use SMT to perform linear search on the loss function

More on SMT

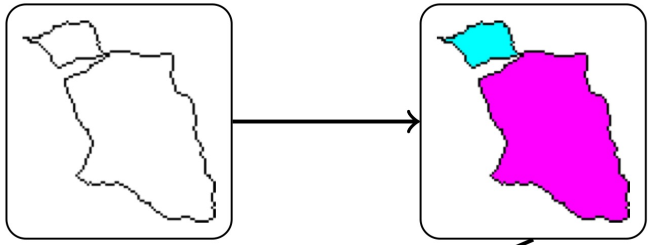
- Satisfiability modulo theories
    - Generalizes SAT such that each clause can contain real numbers, integers, strings, quantifiers …
- Highly expressive, but its solvers only scale under well-defined scenarios
- Representative solver: z3 from Microsoft

Example: Visual Concept Learning

- Space of programs: simple graphic programs that control a turtle
  - Rotations
  - Forward movement
  - Rescaling of shapes

  …

- Program outputs: image parses
  - A list of shapes <id, scale, x, y>
  - A list of containment relationships (i, j)
  - A list of reflexive borders relations borders (i, j)



$$
\begin{aligned}
s_1 &= \mathrm{Shape}(\mathrm{id}=1, \mathrm{scale}=1,\\
&\qquad x=10, y=15)\\
s_2 &= \mathrm{Shape}(\mathrm{id}=2, \mathrm{scale}=1,\\
&\qquad x=27, y=54)\\
&\mathrm{borders}(s_1, s_2)
\end{aligned}
$$

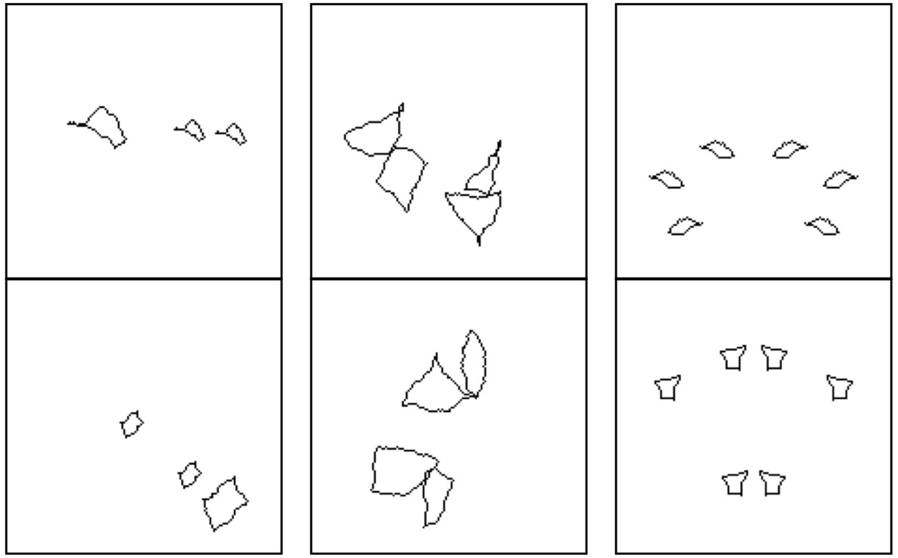
Example: Visual Concept Learning

- Program inputs:
  - Shapes
  - Positions
  - Movement lengths and angles
  - Scales

- A noise model $P_{x|z}(* \mid *)$ that specifies how an output z produces a parse x
  - Positions (add uniform random noise)
  - Optional borders and contains relations are erased with half chance
  - The indices/orders of shapes are randomly permuted

Example: Visual Concept Learning

Example Program



```
teleport(position[0],
           initialOrientation)
draw(shape[0], scale = 1)
move(distance[0], 0deg)
draw(shape[0], scale = scale[0])
move(distance[0], 0deg)
draw(shape[0], scale = scale[0])
```

Conclusion on Ellis et al., 2015

- Manually separated the deterministic part from the probabilistic part

- Convert the problem into an optimization problem by maximizing likelihood and minimizing encoding lengths

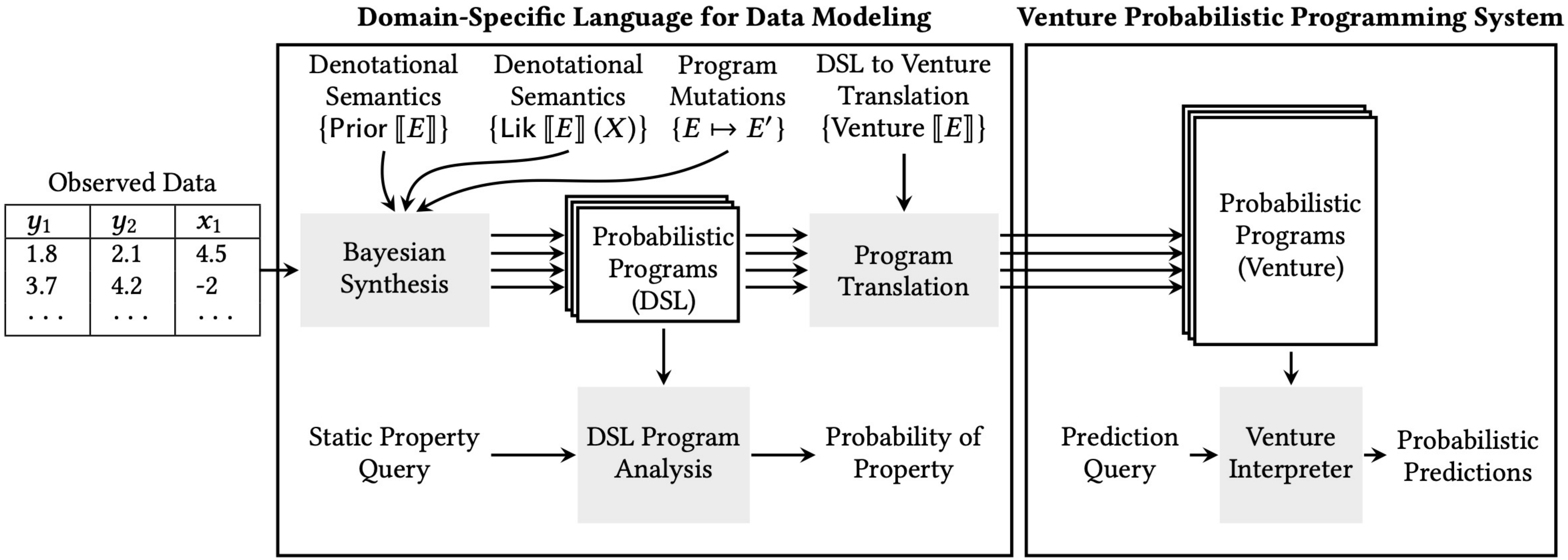
Overview: Saad et al., 2019

- Feras A. Saad, Marco F. Cusumano-Towner, Ulrich Schaechtle, Martin C. Rinard, Vikash K. Mansinghka: Bayesian Synthesis of Probabilistic Programs for Automatic Data Modeling. POPL 19.

- Usage: generate probabilistic programs as generative models of data

- A prior over distribution of programs; conditioning on the observed data, to infer the posterior distribution of the program

Overview of the Framework



From the Original Paper

Details of the Approach

- https://www.youtube.com/watch?v=T5fdUmYJsjM

More on Gaussian Process

- A distribution over functions (from x to y)

- Non-parametric model
  - With infinite many parameters

- The function can be seen as vector which is drawn from a big correlated Gaussian distribution
  - Specified by covariance functions

How to Sample Programs?

- MCMC (Metropolis-Hasting)
- Prior distribution: specified by the PCFG
- Accepting probability: correlates to likelihood

Conclusion on Saad et al., 2019

- A general Bayesian framework to handle different types of synthesis problems
  - Parameterized by the DSL
- Synthesize full programs in Bayesian manner
  - Scalability might be a problem
  - Choosing DSLs and priors are the key

Next Lecture

- Probabilistic Logic Programming