Introduction to Probabilistic Programming

Xin Zhang
Peking University
About the Course

• Part of the international graduate program at PKU

• Offered to both domestic and international students

• Course materials in English

• We do have international students in the class so I will speak in English

• In person, use any language you like
Instructor

• Xin Zhang, Assistant Professor, Computer Science
• Research Areas: Programming Languages, Software Engineering, Assured Artificial Intelligence
• Website: http://xinpl.github.io
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TA

- Ruyi Ji, PhD Student
- Programming Languages Lab @ PKU
- Website: https://jiry17.github.io/
- Email: jiruyi910387714@pku.edu.cn
Logistics

• Time:
  • Thursday 9am - 12am (9:00-9:50, 10:10 – 11:00, 11:10 – 12:00)

• Location:
  • Changping Campus: 211 Teaching Building (昌平校区教学楼211)
  • Main Campus: 311 Teaching Building 2 (主校区2教311)
  • Watch online at using 小鱼易连

• Credit hours: 3

• Course Website: xinpl.github.io/courses/probprog/
Please Provide Feedback

• First semester that we give lectures across campuses
  • So there will be issues
  • Please be patient and give us feedback in time!

第一次实行双校区上课，必然会遇到一些技术挑战，请保持耐心并及时提供反馈
Course WeChat Group
Course Policy

• Laptops and phones allowed to try example programs
  • Mute them
  • Don’t use them to do things that are unrelated to the course

• Academic integrity
  • http://www.dean.pku.edu.cn/web/student_info.php?type=3&id=49
  • 0 tolerance for academic dishonesty and cheating

• Speak out when you don’t understand something
Grading

• Assignments: 30%

• Mid-Term (In-Class Exam): 30%

• Final (In-Class Exam): 40%
What you will learn

• What probabilistic programming is
• How to write programs in popular languages
• Relevant concepts like graphical models, probabilistic inferences
• Theoretical foundations
• Inferences and learning
• Frontiers
Schedule (Tentative)

• Introduction with WebPPL (Week 1 & 2)
• Relevant backgrounds (Week 3 & 4)
• Theoretical foundations (Week 5 & 6)
• Mid-Term (Week 7)
• Inference (Week 8 & 9)
• Learning (Week 10)
• Probabilistic Logic Programming (Week 11 & 12)
• Advanced Topics (Week 13-15)
• Final (Week 16)
After This Class

• Hopefully, you can
  • Write probabilistic programs
  • Have an idea how probabilistic programming languages work underneath
How would you build an AI?
What should it be able to do?
Different Styles of AI

Five Tribes of Machine Learning

Symbolists

Connectionists

Bayesians

Evolutionaries

Analogizers

“The Master Algorithm”, Pedro Domingos
Different Styles of AI

Five Tribes of Machine Learning

Symbolists
Connectionists
Bayesians
Evolutionaries
Analogizers

Probabilistic programming = Programming Languages

“The Master Algorithm”, Pedro Domingos
Spirit of Probabilistic Programming

• Express your beliefs and uncertainties to generate data

• Adjust the model based on observed data

Bayesian
Spirit of Probabilistic Programming

• Express your beliefs and uncertainties to generate data
  If Xiaoming stays up, there is 50% chance he will drink coffee. If he stays up and doesn’t drink coffee, he will fall asleep in class.

• Adjust the model based on observed data
  Given Xiaoming didn’t fall asleep today, how likely did he stay up?

Bayesian
Spirit of Probabilistic Programming

• Express your beliefs and uncertainties to generate data in programs

• Adjust the model based on observed data using general algorithms

Bayesian + Programming Languages
Old Idea that Resurged Recently

• “High level probabilistic languages have been in use since the earliest versions of FORTRAN and BASIC.”- Semantics of Probabilistic Programs, Dexter Kozen, FOCS 1979

• Resurged due to
  • Novel applications
  • New inference algorithms
  • More computing power
General-purpose programming languages + probabilistic constructs
### List of probabilistic programming languages

This article may contain an excessive amount of intricate detail that may interest only a particular audience. Please help by spinning off or relocating any relevant information, and removing excessive detail that may be against Wikipedia's inclusion policy. (October 2019) ([Learn how and when to remove this template message](https://en.wikipedia.org/w/index.php?title=Template:RemoveTemplateMessage/1))

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<thead>
<tr>
<th>Name</th>
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<td><strong>Analytical</strong>[7]</td>
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<td>SQLite, Python</td>
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<td>.NET Framework</td>
<td>.NET Framework</td>
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<td>dimple[10]</td>
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<td>FACTORIE[30]</td>
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<td>PMTK[31]</td>
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<td>Alchemy[32]</td>
<td>C++</td>
<td></td>
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<td>Dyna[33]</td>
<td>Prolog</td>
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<tr>
<td>Figaro[34]</td>
<td>Scala</td>
<td>Scala</td>
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<td>Church[35]</td>
<td>Scheme</td>
<td>Various: JavaScript, Scheme</td>
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<td>ProbLog[36]</td>
<td>Prolog</td>
<td>Python, Jython</td>
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<td>ProBT[37]</td>
<td>C++, Python</td>
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<td>Stan[15]</td>
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<td>C++</td>
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<tr>
<td>Haikuru[38]</td>
<td>Haskell</td>
<td>Haskell</td>
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<tr>
<td>BAli-Phy (software)[39]</td>
<td>Haskell</td>
<td>C++</td>
</tr>
</tbody>
</table>
Probabilistic Programming

• Introduces new machine learning models

• Introduces new programming models
Old-School Bayesian Models

Given the grass is wet, how likely did it rain?

The picture is from Wikipedia.
Inverse Graphics – 3D faces rendered from 2D images using only 50 lines of PPL code.
Old-School Languages vs. PPL

The semantics of the program is defined on a given execution.

Built-in support for random variables and operations on them.

The semantics of the program is defined on a distribution of executions.
First Probabilistic Program in WebPPL

• There is a gacha game:
  • 10% chance to get a SSR card every pull.
  • A pull costs 10 RMB.
  • Xiaoming usually pulls 0-9 times a day.

• How many SSRs can he get a day?

• We can model it using a probabilistic program in WebPPL!

Subset of Javascript + Probabilistic Constructs
var gatcha = function(){
    var pull = Bernoulli({p:0.1});
    var VS = rangeArray(0,10);
    var num_pull = Categorical({vs: VS});
    var num_pull_inst = sample(num_pull);
    var performPull = function(c){
        if( c == 0){
            return 0;
        }
        return sample(pull)+performPull(c-1);
    }
    return performPull(num_pull_inst);
}

gatcha()

- 10% chance to get a SSR card every pull.
- A pull costs 10 RMB.
- Xiaoming usually pulls 0-9 times a day.

You can install WebPPL locally or directly try it at [http://webppl.org](http://webppl.org)!

Builtin distributions/random variables

Let's see it!
var gatcha = function(){
  var pull = Bernoulli({p:0.1});
  var VS = rangeArray(0,10);
  var num_pull = Categorical({vs: VS});
  var num_pull_inst = sample(num_pull);
  var performPull = function(c){
    if( c == 0){
      return 0;
    }
    return sample(pull)+performPull(c-1);
  }
  return performPull(num_pull_inst);
};

gatcha()

• 10% chance to get a SSR card every pull.
• A pull costs 10 RMB.
• Xiaoming usually pulls 0-9 times a day.

• What if I want to know the average number of SSR Xiaoming gets?
• What if I want to know what is the chance that Xiaoming gets two SSRs?
• What if I want to know the full distribution?

Well, you can do it manually, but WebPPL has built-in support for these!
• 10% chance to get a SSR card every pull.
• A pull costs 10 RMB.
• Xiaoming usually pulls 0-9 times a day.

• What if I want to know the average number of SSR Xiaoming gets?
• What if I want to know what is the chance that Xiaoming gets two SSRs?
• What if I want to know the full distribution?

```javascript
var gatcha = function() {
    var pull = Bernoulli({p: 0.1});
    var VS = rangeArray(0, 10);
    var num_pull = Categorical({vs: VS});
    var num_pull_inst = sample(num_pull);
    var performPull = function(c){
        if (c == 0) {
            return 0;
        } else {
            return sample(pull) + performPull(c - 1);
        }
        return performPull(num_pull_inst);
    };
    return performPull(num_pull_inst);
};

var gatcha_model = Infer({model: gatcha});
display(expectation(gatcha_model))
display(Math.exp(gatcha_model.score(3)))
viz(gatcha_model)
```

• Built-in distributions/random variables
• 10% chance to get a SSR card every pull.
• A pull costs 10 RMB.
• Xiaoming usually pulls 0-9 times a day.

“Black magic” behind this function invocation. It performs **Marginal Inference**, which creates a map from values to probabilities.

```javascript
var gatcha = function(){
    var pull = Bernoulli({p:0.1});
    var VS = rangeArray(0,10);
    var num_pull = Categorical({vs: VS});
    var num_pull_inst = sample(num_pull);
    var performPull = function(c){
        if( c == 0){
            return 0;
        }
        return sample(pull)+performPull(c-1);
    }
    return performPull(num_pull_inst);
};
var gatcha_model = Infer({model: gatcha})
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```

Built-in distributions/random variables

• What if I want to know the average number of SSR Xiaoming gets?
• What if I want to know what is the chance that Xiaoming gets two SSRs?
• What if I want to know the full distribution?

“Black magic” behind this function invocation. It performs **Marginal Inference**, which creates a map from values to probabilities.
More on Marginal Probabilities

• We can enumerate all possible worlds and calculate their probabilities.

• Marginal probability of $x = X$ is defined as

$$P(x = X) = \Sigma_{w|w \{x=x\ \text{in} \ w} P(w)$$

We will talk about how to calculate them later in the course.
Let’s make the problem more interesting
• 10% chance to get a SSR card every pull.
• A pull costs 10 RMB.
• **If 5 continuous pulls yield 0 SSR, the 6th pull guarantees an SSR.**
• Xiaoming usually pulls 0-9 times a day.

```javascript
var gatcha = function(){
  var pull = Bernoulli({p:0.1});
  var VS = rangeArray(0,10);
  var num_pull = Categorical({vs: VS});
  var num_pull_inst = sample(num_pull);
  var performPull = function(c){
    if( c == 0){
      return 0;
    }
    return sample(pull)+performPull(c-1);
  }
  return performPull(num_pull_inst)
};

var gatcha_model = Infer({model: gatcha})

display(expectation(gatcha_model))
display(Math.exp(gatcha_model.score(3)))
viz(gatcha_model)
```
var gatcha = function(){
  var pull = Bernoulli({p:0.1});
  var VS = rangeArray(0,10);
  var num_pull = Categorical({vs: VS});
  var num_pull_inst = sample(num_pull);
  var performPull = function(c, num_no_ssr){
    if( c == 0){
      return 0;
    }
    if (num_no_ssr== 5){
      return 1 + performPull(c-1, 0);
    }
    else{
      var cp = sample(pull);
      if (cp)
        return 1 + performPull(c-1, 0);
      else
        return 0 + performPull(c-1, num_no_ssr+1);
    }
  }
  return performPull(num_pull_inst, 0);
};

• 10% chance to get a SSR card every pull.
• A pull costs 10 RMB.
• If 5 continuous pulls yield 0 SSR, the 6th pull guarantees an SSR.
• Xiaoming usually pulls 0-9 times a day.

• What if I want to know the average number of SSR Xiaoming gets?
• What if I want to know what is the chance that Xiaoming gets three SSRs?
• What if I want to know the full distribution?

Xin Zhang@PKU
Informal Semantics of Probabilistic Programs

- Probabilistic programs define a distribution of executions
  - The randomness comes from random variables
  - Given an execution $e$, let $V$ be the set of random variables that are evaluated in $e$, we use $v(e)$ to denote the value a random variable $v$ takes in $e$, then we have
    \[ P(e) = \prod_{v \in V} P(v = v(e)) \]
  - We will introduce formal semantics later in the course!
• 10% chance to get a SSR card every pull.
• A pull costs 10 RMB.
• If 5 continuous pulls yield 0 SSR, the 6th pull guarantees an SSR.
• Xiaoming usually pulls 0-9 times a day.

Hey! What you assume seems quite reasonable. But Xiaoming never gets more than two SSRs a day!
var gatcha = function() {
  var pull = Bernoulli({p: 0.1});
  var VS = rangeArray(0, 10);
  var num_pull = Categorical({vs: VS});
  var num_pull_inst = sample(num_pull);
  var performPull = function(c, num_no_ssr) {
    if (c == 0) {
      return 0;
    }
    if (num_no_ssr == 5) {
      return 1 + performPull(c - 1, 0);
    }
    else {
      var cp = sample(pull);
      if (cp) {
        return 1 + performPull(c - 1, 0);
      } else {
        return 0 + performPull(c - 1, num_no_ssr + 1);
      }
    }
  }
  return performPull(num_pull_inst);
};

- 10% chance to get a SSR card every pull.
- A pull costs 10 RMB.
- If 5 continuous pulls yield 0 SSR, the 6th pull guarantees an SSR.
- Xiaoming usually pulls 0-9 times a day.
- **Xiaoming never gets more than 2 SSRs a day.**

This describes the outcome rather than the process. How should we modify the program? **New Language Construct!**
• 10% chance to get a SSR card every pull.
• A pull costs 10 RMB.
• If 5 continuous pulls yield 0 SSR, the 6th pull guarantees an SSR.
• Xiaoming usually pulls 0-9 times a day.
• **Xiaoming never gets more than 2 SSRs a day.**

This describes the outcome rather than the process. How should we modify the program?

```javascript
var gatcha = function(){
  ...
  return performPull(num_pull_inst)
};

var gatcha1 = function(){
  var num_ssrs = gatcha()
  condition(num_ssrs <= 2)
  return num_ssrs
}

var gatcha_model = Infer({model: gatcha1})

display(expectation(gatcha_model))
display(Math.exp(gatcha_model.score(3)))
viz(gatcha_model)
```

Filter out executions that do not satisfy external knowledge, and computes a conditional distribution.
More on **Condition**

var d = sample(Categorical({ps: [0.2, 0.3, 0.5], vs: [1,2,3]}))

condition(d != 3)

<table>
<thead>
<tr>
<th>Value of d</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.2</td>
</tr>
<tr>
<td>2</td>
<td>0.3</td>
</tr>
<tr>
<td>3</td>
<td>0.5</td>
</tr>
</tbody>
</table>
More on **Condition**

```javascript
var d = sample(Categorical({ps: [0.2, 0.3, 0.5], vs: [1,2,3]}))
condition(d != 3)
```

<table>
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<td>0.3</td>
</tr>
<tr>
<td>3</td>
<td>0.5</td>
</tr>
</tbody>
</table>
More on Condition

```
var d = sample(Categorical({ps: [0.2, 0.3, 0.5], vs: [1,2,3]}))
```

c\text{condition}(d \neq 3)

<table>
<thead>
<tr>
<th>Value of d</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$0.2 / (0.2+0.3)$</td>
</tr>
<tr>
<td>2</td>
<td>$0.3 / (0.2+.3)$</td>
</tr>
<tr>
<td>3</td>
<td>0.5</td>
</tr>
</tbody>
</table>
More on **Condition**

```javascript
var d = sample(Categorical({ps: [0.2, 0.3, 0.5], vs: [1,2,3]}))

condition(d != 3)
```

<table>
<thead>
<tr>
<th>Value of d</th>
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<td>0.4</td>
</tr>
<tr>
<td>2</td>
<td>0.6</td>
</tr>
<tr>
<td>3</td>
<td>0.5</td>
</tr>
</tbody>
</table>
More on **Condition**

• Recall our informal semantics

\[ P(e) = \prod_{v \in V} P(v = v(e)) \]

• We use \( P(e|c) \) to denote the probability of an execution \( e \) after a condition statement \( c \) is added to the program, and we use \( c(e) \) to represent whether \( c \) holds on \( e \), then

\[
P(e|c) = \begin{cases} 
0, & \text{if } c(e) = \text{false} \\
\frac{P(e)}{\sum_{e' \in \{e''|c(e'')\}} P(e')}, & \text{if } c(e) = \text{true}
\end{cases}
\]
More on Condition

• It might be obvious, but condition does construct a conditional distribution:

\[
P(e|c) = \frac{P(e \cap c)}{P(c)} = \frac{P(e \cap c)}{\sum_{e' \in E} P(c \cap e')} = \frac{P(c|e) \cdot P(e)}{\sum_{e' \in E} P(c|e') \cdot P(e')} = \begin{cases} 
0, & \text{if } c(e) = false \\
\frac{P(e)}{\sum_{e'' \in \{e''|c(e'')\}} P(e')}, & \text{if } c(e) = true
\end{cases}
\]
Main Probabilistic Constructs in WebPPL

• Built-in random variables and sample together describe a random process in a constructive way

• Condition provides a way to incorporate external knowledge about the output

They together provide a mechanism to do Bayesian learning

What you believe + What you observe
A Simple Bayesian Inference Example

• A disease can happen to 1% of the population
• A test method has the following property:

<table>
<thead>
<tr>
<th></th>
<th>Tested Positive</th>
<th>Tested Negative</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actually Positive</td>
<td>90%</td>
<td>10%</td>
</tr>
<tr>
<td>Actually Negative</td>
<td>10%</td>
<td>90%</td>
</tr>
</tbody>
</table>

• Xiaoming is tested positive. How likely does he carry the disease?
A Simple Bayesian Inference Example

• We can calculate the probability using Bayes’ theorem

\[
P(\text{positive } | \text{tested positive}) = P(\text{positive}) \times \frac{P(\text{tested positive} | \text{positive})}{P(\text{tested positive})}
\]

\[
= 0.01 \times \frac{0.9}{0.01 \times 0.9 + 0.99 \times 0.1}
\]

\[
= 0.083
\]
A Simple Bayesian Inference Example

• The probabilistic program is as follow:

```javascript
var medic_test = function(){
    var positive = sample(Bernoulli({p:0.01}))
    var test_positive = function(pg){
        if(pg){
            return sample(Bernoulli({p:0.9}))
        }
        else
            return !sample(Bernoulli({p:0.9}))
    }
    condition(test_positive(positive))
    return positive
}

var m = Infer({model:medic_test})
display(Math.exp(m.score(true)))
viz(m)
```
• 10% chance to get a SSR card every pull.
• A pull costs 10 RMB.
• If 5 continuous pulls yield 0 SSR, the 6th pull guarantees an SSR.
• 20% of players believe the rate is not as advertised, but only 8%.
• To test if the assumption is true, Xiaoming pulled 20 times, and got 4 SSRs.

What is the chance that the rate is actually 8%?

Try to calculate it using Bayes’ theorem!
• 10% chance to get a SSR card every pull.
• A pull costs 10 RMB.
• If 5 continuous pulls yield 0 SSR, the 6th pull guarantees an SSR.
• 20% of players believe the rate is not as advertised, but only 8%.
• To test if the assumption is true, Xiaoming pulled 20 times, and got 4 SSRs.

In WebPPL, it is easy!

```javascript
var gatcha = function(){
  var cheated = sample(Bernoulli({p:0.2}));
  var pull = function(){
    if (cheated){
      return Bernoulli({p:0.08});
    }
    else
      return Bernoulli({p:0.1});
  }
  var num_pull_inst = 20;
  var performPull = function(c, num_no_ssr){
    ...
  }
  var num_ssrs = performPull(num_pull_inst, 0)
  condition(num_ssrs == 4)
  return cheated;
};

var gatcha_model = Infer({model: gatcha})
display(expectation(gatcha_model))
```
• 10% chance to get a SSR card every pull.
• A pull costs 10 RMB.
• If 5 continuous pulls yield 0 SSR, the 6th pull guarantees an SSR.
• **When spending over 1000 RMB, for every new pull, there is a chance that the bank would call Xiaoming.**
• Suppose the chance is \((x-1000)/1000\), and Xiaoming got called and then stopped, how many SSRs has he pulled so far?
• 10% chance to get a SSR card every pull.
• A pull costs 10 RMB.
• If 5 continuous pulls yield 0 SSR, the 6th pull guarantees an SSR.
• When spending over 1000 RMB, for every new pull, there is a chance that the bank would call Xiaoming.
• Suppose the chance is \((x-1000)/1000\), and Xiaoming got called and then stopped, how many SSRs has he pulled so far?

Can you express this problem using a conventional graphical model like a Bayesian network?

```javascript
var gatcha = function(){
...

var VS = rangeArray(0, 1000);
var num_pull = Categorical({vs: VS});
...

var performPull = function(c, num_no_ssr){
    if( c == 0){
        return [0, false];
    }
    var cost = (num_pull_inst - c) * 10;
    if(cost > 2000)
        return [0, true]
    if(cost > 1000)
        if(sample(Bernoulli({p: (cost - 1000)/1000.0}))){
            return [0, true]
        }
    ...
}

var pull_result = performPull(num_pull_inst, 0)
condition(pull_result[1])
return pull_result[0]
};
```
Probabilistic Programming So Far

• Built-in support for random variables
  • Categorical, Bernoulli ….
  • Sample

• A general language to describe the sampling process
  • Subset of Javascript

• The ability to impose conditions on any state
  • Condition
Probabilistic Programming So Far

• Built-in support for random variables
  • Categorical, Bernoulli ….
  • Sample

• A general language to describe the sampling process
  • Subset of Javascript

• The ability to impose conditions on any state
  • Condition

A convenient way to express highly complex distributions

Don’t worry about how to calculate it! Just think about what is it!
That sounds too good to be true. There must be a catch here ...
That sounds too good to be true. There must be a catch here ...

Well, some of the probabilistic programs can be really slow to run.

People have been working on how to make probabilistic programs run fast, which we will discuss later in the course.
var gatcha = function(){
    var pull = Bernoulli({p:0.1});
    var VS = rangeArray(0,1000);
    var num_pull = Categorical({vs: VS});
    var num_pull_inst = sample(num_pull);
    var performPull = function(c, num_no_ssr){
        if( c == 0){
            return 0;
        }
        if (num_no_ssr == 5){
            return 1 + performPull(c-1, 0);
        }
        else{
            var cp = sample(pull);
            if (cp)
                return 1 + performPull(c-1, 0);
            else
                return 0 + performPull(c-1, num_no_ssr+1);
        }
    }
    return performPull(num_pull_inst)
};

var gatcha1 = function(){
    var num_ssrs = gatcha()
    condition(num_ssrs <= 2)
    return num_ssrs
}

var gatcha_model = Infer({model: gatcha1})

display(expectation(gatcha_model))
viz(gatcha_model)

• 10% chance to get a SSR card every pull.
• A pull costs 10 RMB.
• If 5 continuous pulls yield 0 SSR, the 6th pull guarantees an SSR.
• Xiaoming usually pulls 0-999 times a day.
• Xiaoming never gets more than 2 SSRs a day.
Next Class

- More about WebPPL
  - We have talked about the core probabilistic constructs

- Representative applications using WebPPL