Probabilistic Logic Programming

Xin Zhang
Peking University
Recap of Last Lecture

• Learning in probabilistic programming
  • Parameter learning, structure learning
  • Still an active research area
This Lecture

• Probabilistic logic programming
  • Motivation
  • Syntax
  • Semantics
  • Inference
Classical AI: Logic

• Rich logic systems provide significant expressiveness power
  • Concise and learnable models

• Example: first-order logic. Rules of chess occupy
  • $10^0$ pages of first-order logic
  • $10^5$ pages in propositional logic
  • $10^{38}$ pages in finite automata
Quick Recap on First-Order Logic

• Compared to propositional logic, introduces predicates and quantifications for expressiveness

\[ \forall h_1, h_2, h_3. \text{sibling}(h_1, h_2) \land \text{sibling}(h_2, h_3) \rightarrow \text{sibling}(h_1, h_3) \]

• Undecidable
Modern AI: Probability Theory for Uncertainty

• Bayesian network

• Fixed variables in fixed ranges
  • Similar to propositional logic and Boolean logic
Probabilistic Logic Programming: Unifying Logic and Probability

• Logic: the ability to describe complex domains concisely in terms of objects and relations

• Probability: the ability to handle uncertainty

• Logic + probability = Probabilistic Logic Programming
Example Probabilistic Logic Languages

- Markov Logic Network. University of Washington
- Probabilistic Soft Logic. University of Maryland
- **Problog**. KU Leuven  
- BLOG. UC Berkeley
- ....
Background: Logic Programming

• Declarative: specifies what rather than how

• Leverages powerful inference engine
Background: Prolog and Datalog

- Prolog: once popular in AI, still being used in pattern matching (NLP)
  - Turning-complete

- Datalog: a subset of Prolog
  - Can only express polynomial algorithms
  - Originates from the Database community (SQL with recursions)
  - Logic part of Problog
Background: Datalog

Input Relation:
Edge(e1, e2)

Output Relation:
Path(e1, e2)

Rules:
Path(e1, e2) :- edge(e1, e2)  \quad \forall \text{edge}(e_1, e_2) \Rightarrow \text{path}(e_1, e_2)
Path(e1, e3) :- path(e1, e2), edge(e2, e3) \quad \forall \text{path}(e_1, e_2) \land \text{edge}(e_2, e_3) \Rightarrow \text{path}(e_1, e_2)
Background: Datalog

Edge(1, 2)       Edge(2, 3)

Path(1, 2) :- Edge(1, 2)
Path(2, 3) :- Edge(2, 3)

Path(1, 3) :- Path(1, 2), Edge(2, 3)
Adding Probabilities to Datalog

• If A is a friend of B, and B is a friend of C, then A is likely a friend of C.

Can you write a program for the above sentence?
Adding Probabilities to Datalog

• Suppose edges exist with probabilities (by observation), compute path reachability.

Can you write a program for the above sentence?

Add probabilities to rules or facts?
What is the semantics?

Path(E1, E2) :- edge(E1, E2)
0.5: Path(E1, E3) :- path(E1, E2), edge(E2, E3)

Given a set of derived tuples/facts, assign a probability to them.
Problog: Introduction

• A language developed by the group led by Luc De Raedt at KU Leuven

• Extends Prolog with probabilities
  • Actually closer to Datalog
Problog: Syntax

• **Value**: numbers, mixed numbers and letters starting with a letter in lower cases

• **Variable**: starting with a capital letter
## Problog: Syntax

<table>
<thead>
<tr>
<th>Definition</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>fact</td>
<td>a.</td>
</tr>
<tr>
<td>probabilistic fact</td>
<td>0.5::a.</td>
</tr>
<tr>
<td>clause</td>
<td>a :- x.</td>
</tr>
<tr>
<td>probabilistic clause</td>
<td>0.5::a :- x.</td>
</tr>
<tr>
<td>annotated disjunction</td>
<td>0.5::a; 0.5::b.</td>
</tr>
<tr>
<td>annotated disjunction</td>
<td>0.5::a; 0.5::b :- x.</td>
</tr>
</tbody>
</table>

From the documentation of Problog
Problog: Syntax

Queries:

\[
0.5::\text{heads}(C). \\
two\_heads \leftarrow \text{heads}(c1), \text{heads}(c2). \\
\text{query}(two\_heads).
\]

\[
0.5::\text{heads}(C) \leftarrow \text{between}(1, 4, C). \\
\text{query}(\text{heads}(C)).
\]

\[
0.5::\text{heads}(C) \leftarrow \text{between}(1, 4, C). \\
\text{query}(\text{heads}(C)).
\]

Evidence:

\[
0.5::\text{heads}(C). \\
two\_heads \leftarrow \text{heads}(c1), \text{heads}(c2). \\
\text{evidence}(\text{\textbackslash + two\_heads}). \\
\text{query}(\text{heads}(c1)).
\]

From the documentation of Problog
Example Program I

0.5 :: stayUp.
0.7 :: drinkCoffee :- stayUp.
0.5 :: drinkCoffee :- \+ stayUp.
0.9 :: fallSleep :- \+ drinkCoffee, stayUp.
0.3 :: fallSleep :- drinkCoffee, stayUp.
0.1 :: fallSleep :- \+ stayUp.

evidence(fallSleep).

query(stayUp).
What does the following program compute?

0.5 :: stayUp.
0.7 :: drinkCoffee :- stayUp.
0.5 :: drinkCoffee :- \\+ stayUp.
0.9 :: fallSleep :- \\+ drinkCoffee, stayUp.
0.3 :: fallSleep :- drinkCoffee, stayUp.
0.1 :: fallSleep :- \\+stayUp.

query(stayUp).

evidence(fallSleep).
What does the following program compute?

0.5::heads1.
0.5::heads2.

heads1 :- heads2.

query(heads1).
query(heads2).

What does the following program compute?

0.5::heads1.
0.5::heads2.

\(+\) heads1 :- heads2.

query(heads1).
query(heads2).
Example Program 2

0.9 :: edge(0,1).
0.8 :: edge(1,2).
0.7 :: edge(2,3).
0.8 :: edge(2,4).

1 :: path(A,B) :- edge(A,B).
0.8 :: path(A,C) :- path(A,B), edge(B,C).

evidence(+ path(0,3)).

query(path(0,4)).
Semantics of Problog

• What is the semantics of the following program?
  0.5 :: stayUp.
  0.7 :: drinkCoffee :- stayUp.
  0.3 :: fallSleep :- drinkCoffee, stayUp.

query(fallSleep).
Semantics of Problog

• For simplicity, we assume all probabilities are attached to facts

• First idea: we can convert the program into a Bayesian network, but how?
Semantics of Problog

• Converting into a Bayesian network is viable, but there are small catches

• We give another semantics that defines a distribution of Datalog programs
Semantics of Problog

• From a Problog program, we can sample a Datalog program by sampling the facts

\[
\begin{align*}
0.5 & :: \text{stayUp}.
0.7 & :: \text{r1}.
0.3 & :: \text{r2}.
\text{drinkCoffee} & :- \text{stayUp}, \text{r1}.
\text{fallSleep} & :- \text{drinkCoffee}, \text{stayUp}, \text{r2}.
\end{align*}
\]

\[
= \quad \text{sample}
\]

\[
\begin{align*}
\text{stayUp}. & \\
\text{r1}. & \\
\text{r2}. & \\
\text{drinkCoffee} & :- \text{stayUp}, \text{r1}.
\text{fallSleep} & :- \text{drinkCoffee}, \text{stayUp}, \text{r2}.
\end{align*}
\]

Probability: 0.5*0.7*0.3
Semantics of Problog

• What about queries?

0.5 :: stayUp.
0.7 :: r1.
0.3 :: r2.
drinkCoffee :- stayUp, r1.
fallSleep :- drinkCoffee, stayUp, r2.

query(fallSleep)

A query calculates a marginal probability of a fact. Informally,

\[
p(f) = \frac{\sum p(\text{any program that derives } f)}{\sum p(\text{any program})}
\]
Semantics of Problog

• What about evidence?

0.5 :: stayUp.
0.7 :: r1.
0.3 :: r2.
drinkCoffee :- stayUp, r1.
fallSleep :- drinkCoffee, stayUp, r2.

evidence(\+ fallSleep)
query(stayUp)

Evidence filters out certain programs. Informally,

\[ p(f) = \frac{\sum p(\text{any program that derives } f|\text{evidence})}{\sum p(\text{any program}|\text{evidence})} \]
Semantics of Problog

• What about relations and quantified variables?

0.9 :: edge(0,1).
0.8 :: edge(1,2).
0.7 :: edge(2,3).
0.8 :: edge(2,4).

path(A,B) :- edge(A,B).
0.8 :: path(A,C) :- path(A,B), edge(B,C).

evidence(\ + path(0,3)).

query(path(0,4)).
Semantics of Problog

• Move probabilities to facts

0.9 :: edge(0,1).
0.8 :: edge(1,2).
0.7 :: edge(2,3).
0.8 :: edge(2,4).
0.8 :: r(A,B,C).

path(A,B) :- edge(A,B).
path(A,C) :- path(A,B), edge(B,C), r(A,B,C).

evidence(\ + path(0,3)).

query(path(0,4)).
Semantics of Problog

• Ground

Constants: 0, 1, 2, 3 4

path(A,C) :- path(A,B), edge(B,C), r(A,B,C).

Generates

path(0,0) :- path(0,0), edge(0,0), r(0,0,0). A=0, B=0, C=0
path(0,1) :- path(0,0), edge(0,1), r(0,0,1). A=0, B=0, C=1
path(0,1) :- path(0,0), edge(0,1), r(0,0,1). A=0, B=0, C=1
...
Semantics of Problog

• After grounding, each ground term can be seen as a Boolean variable, then the whole program can be solved using the semantics of the Boolean case

```
path(0,0) :- path(0,0), edge(0,0), r(0,0,0).

t1 :- t1,t2,t3
```
Semantics of Problog

• First, ground the program into a Boolean program

• The Boolean program describes a distribution of Datalog program, which in turn defines a distribution of outputs
Questions

• Can you use Problog to express uniform distributions?

• What about loops?
Logic Part in Problog is more than Datalog

:- use_module(library(aggregate)).

pull(0).
count(1).

pull(N+1) :- pull(N), N < 10.
0.1 :: pull_SSR(N) :- pull(N).

num_SSRs(sum<X>) :- pull_SSR(N),count(X).

query(num_SSRs(X)).
But It is also Not Prolog

• The following program terminates in Problog but not in Prolog

child(anne,bridget).
child(bridget,caroline).
child(caroline,donna).
child(donna,emily).
descend(X,Y) :- descend(Z,Y), child(X,Z).
descend(X,Y) :- child(X,Y).
query(descend(anne,emily))
Inference

• As described before, inference can be done in two steps:
  • **Grounding.** Convert the program into a probabilistic program with only Boolean variables (no quantifiers)
  • **Solving.** Solve with the Boolean program produced above.
Optimization on Grounding

• Grounding replaces all variables with their values
  • Number of grounded rules is proportional to cartesian product of the domain sizes

• How to optimize?
  • A simple idea: only ground the part that is relevant to the queries and evidence.
  • Backtrack over the rules starting from the queries and evidence (SLD resolution).
  • A further optimization: stop tracking if a rule body doesn’t hold according to the evidence
Optimization on Grounding

• If the logic part is Datalog without negation, we can use a Datalog solver to compute the grounding

• Datalog without negation is monotonic: the more rules or input facts, the more output facts

• If negation is on the input, it is still fine
Negation in Problog

• Unfortunately, Problog allows the following program:
  one(1).
  odd(X) :- one(X).
  even(X) :- \+ odd(X).

And
  0.5::a. 0.9 :: e:-a.
  0.5::b. 0.9 :: e:-b.
  0.1 :: e:-a,b

If such negations are not present, we can use a Datalog solver to ground, which is highly efficient.
Solving

• Once we have a grounded program, we can leverage existing techniques

• Idea 1: convert the program into a Bayesian network

• Idea 2: convert the program into a Boolean formula with weights (MaxSAT)
Solving: Converting into a Bayesian Net

0.8 :: a.
0.7 :: b.

0.5 :: c:- a.
0.5 :: c:- b.

query(c).
Solving: Converting into a Bayesian Net

• We move all probabilities to input facts
• We add a root node whose prior distribution is P(r =1) = 1. Then we add a rule p :: f:-r for each input fact p::f
• For each fact f, suppose it is derived using r1, …, rn, we add arcs from all facts in the rule bodies to f.
• We set conditional probabilities:
  \[ p(f \mid \vee body(r_i) = True) = 1 \]
  \[ p(f \mid \vee body(r_i) = False) = 0 \]

Only works for program without cycles
Solving: Converting into a MaxSAT

• Finding the most likely solution becomes solving the MaxSAT

• Computing marginal probabilities becomes weighted model counting
Brief Introduction on MaxSAT

MaxSAT:

\[
\begin{align*}
    a & \land \quad (C1) \\
    \neg a & \lor b \land \quad (C2) \\
    4 & \quad \vdash \quad (C3) \\
    2 & \quad \vdash \quad (C4) \\
    7 & \quad \vdash \quad (C5)
\end{align*}
\]

\[\text{Subject to} \quad C1 \quad \text{C2}\]

Maximize \[4 \times C3 + 2 \times C4 + 7 \times C5\]

Solution: \(a = \text{true}, \ b = \text{true}, \ c = \text{true}, \ d = \text{false}\)

(Objective = \(11\))
Brief Introduction on Weighted Model Counting

• Model counting: compute the number of assignments to a SAT expression
  
a or b 3 assignments

• Weighted model counting
  • Each variable has a weight for each assignment: w(v)
  • The model weight is the product of variable weights
  • Now the count is a weighted sum
Example using MaxSAT for Inference

0.6 :: rain.
0.5 :: sprinkle.
0.9 :: grass_wet :- rain, sprinkle.

grass_wet :- rain, sprinkle is translated into

\[ \text{grass_wet} \leftrightarrow \text{rain} \land \text{sprinkle} \]
Example using MaxSAT for Inference

• When translating rules, we have to consider the least fixed point semantics of Datalog

• Suppose the rules are acyclic, for a given fact $f$, we have to consider all grounded rules that derive $f$

$$f \leftrightarrow \lor body(r_i)$$
Example using MaxSAT for Inference

• When rules are cyclic, problems become complicated:

  0.5::a. b:-a. b:-c. c:-b

• For reference:
Example using WMC for Inference

0.6 rain
0.4 !rain
0.5 sprinkle
0.5 !sprinkle
0.9 r
0.1 !r

grass_wet or !rain or !sprinkle
!grass_wet or rain
!grass_wet or sprinkle

\[
w(\text{rain = true}) = 0.6 \\
w(\text{rain = false}) = 0.4 \\
w(\text{sprinkle = true}) = 0.5 \\
w(\text{sprinkle = false}) = 0.5 \\
w(r = \text{true}) = 0.9 \\
w(r = \text{false}) = 0.1
\]

\[P(\text{grass_wet = true}) = \text{WMC}(\text{MA} \text{grass_wet = true})\]

What if we want to evaluate

\[P(\text{rain } | \text{ grass_wet = true})?\]
Using WMC for Marginal Inference

• Let the constructed weighted formula be $M$, queries be $Q$, evidence be $E$, then

$$P(Q) = \frac{WMC(M \land Q \land E)}{WMC(M \land E)}$$

• For more, refer to

Further Reading on Problog

Next Lecture

• Causality